

5-4 Multiplying Binomials Mentally

Objective: To find the product of two binomials mentally.

Vocabulary

Quadratic Term A term of degree two. For example, $2x^2$.

Linear Term A term of degree one. For example, $5x$.

Quadratic Polynomial A polynomial whose term of greatest degree is quadratic. For example, $2x^2 - 5x + 7$.

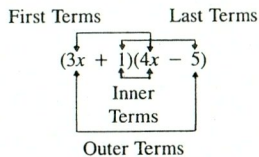
Example 1 Write $(3x + 1)(4x - 5)$ as a trinomial.

Solution 1 You can work horizontally as shown at the left or vertically as shown at the right.

$$\begin{array}{r}
 (3x + 1)(4x - 5) = 3x(4x - 5) + 1(4x - 5) \\
 = 12x^2 - 15x + 4x - 5 \\
 = 12x^2 - 11x - 5
 \end{array}
 \qquad
 \begin{array}{r}
 4x - 5 \\
 3x + 1 \\
 \hline
 12x^2 - 15x \\
 + 4x - 5 \\
 \hline
 12x^2 - 11x - 5
 \end{array}$$

Solution 2 Use the FOIL method to multiply in your head.

Think of the products of these terms:



Then write the products:

$$\begin{array}{ccccccc}
 12x^2 & - & 15x & + & 4x & - & 5 \\
 \text{First} & & \text{Outer} & & \text{Inner} & & \text{Last} \\
 \text{terms} & & \text{terms} & & \text{terms} & & \text{terms} \\
 \hline
 & & & & & & = 12x^2 - 11x - 5
 \end{array}$$

Write each product as a trinomial.

- $(x + 6)(x + 1) x^2 + 7x + 6$
- $(y + 3)(y + 4) y^2 + 7y + 12$
- $(a - 4)(a - 2) a^2 - 6a + 8$
- $(x - 5)(x - 6) x^2 - 11x + 30$
- $(c + 2)(c + 6) c^2 + 8c + 12$
- $(k - 3)(k - 6) k^2 - 9k + 18$
- $(a - 3)(a - 7) a^2 - 10a + 21$
- $(2 + x)(3 + x) 6 + 5x + x^2$
- $(k - 4)(k - 7) k^2 - 11k + 28$
- $(b - 2)(b + 7) b^2 + 5b - 14$
- $(c - 6)(c + 7) c^2 + c - 42$
- $(a - 4)(a - 6) a^2 - 10a + 24$
- $(2a + 3)(a + 4) 2a^2 + 11a + 12$
- $(3x + 2)(x + 4) 3x^2 + 14x + 8$

5-4 Multiplying Binomials Mentally (continued)

Write each product as a trinomial.

- $(2a + 7)(a - 2) 2a^2 + 3a - 14$
- $(4a - 1)(3a - 1) 12a^2 - 7a + 1$
- $(3a - 5)(2a - 1) 6a^2 - 13a + 5$
- $(3 - 2a)(2 - 3a) 6 - 13a + 6a^2$
- $(2k + 1)(3k + 4) 6k^2 + 11k + 4$
- $(3x - 2)(x + 5) 3x^2 + 13x - 10$
- $(4x + 3)(2x - 1) 8x^2 + 2x - 3$
- $(7m - 3)(6m + 2) 42m^2 - 4m - 6$

Example 2 Write $(3x - 4y)(5x + y)$ as a trinomial.

Solution

$$\begin{array}{r}
 \begin{array}{c} \text{F} \quad \text{O} \quad \text{I} \quad \text{L} \\
 (3x - 4y)(5x + y) = 15x^2 + (3xy - 20xy) - 4y^2 \\
 = 15x^2 - 17xy - 4y^2 \end{array}
 \end{array}$$

Write each product as a trinomial.

- $(a - 2b)(a + b) a^2 - ab - 2b^2$
- $(x + 3y)(x + 2y) x^2 + 5xy + 6y^2$
- $(2x + y)(3x - 2y) 6x^2 - xy - 2y^2$
- $(3x + y)(x + 2y) 3x^2 + 7xy + 2y^2$
- $(4x + y)(2x - 3y) 8x^2 - 10xy - 3y^2$
- $(6a - b)(5a - 2b) 30a^2 - 17ab + 2b^2$

Example 3 Write $(m^2 - 3m)(2m^2 + 5m)$ as a trinomial.

Solution

$$\begin{array}{r}
 \begin{array}{c} \text{F} \quad \text{O} \quad \text{I} \quad \text{L} \\
 (m^2 - 3m)(2m^2 + 5m) = m^2(2m^2) + m^2(5m) + (-3m)(2m^2) + (-3m)(5m) \\
 = 2m^4 + (5m^3 - 6m^3) - 15m^2 \\
 = 2m^4 - m^3 - 15m^2 \end{array}
 \end{array}$$

Write each product as a trinomial.

- $(x^2 - 2x)(2x^2 + 3x) 2x^4 - x^3 - 6x^2$
- $(a^2 - 3b)(2a^2 + b) 2a^4 - 5a^2b - 3b^2$
- $(u^2 + v^2)(u^2 - 4v^2) u^4 - 3u^2v^2 - 4v^4$
- $(a^3 - 3b^3)(a^3 + 4b^3) a^6 + a^3b^3 - 12b^6$
- $(x^3 - 2x)(x^3 + 1) x^6 - 2x^4 + x^3 - 2x$
- $(x^3 - y^2)(3x^3 + y^2) 3x^6 - 2x^3y^2 - y^4$

Mixed Review Exercises

Simplify.

- $(3x^2y)(-5x^2y^3) - 15x^4y^4$
- $(6x^2y^4)^3 216x^6y^{12}$
- $6n^3 + 11n^2 - n - 6$
- $\frac{15r^2 + 20r - 25}{5} 3r^2 + 4r - 5$
- $\frac{(4y)^3}{4y} 16y^2$
- $\frac{12 - 6x - 2x^2}{2} 6 - 3x - x^2$

Solve.

- $n = 32 - 3n \{8\}$
- $3x - (2x + 7) = 7 \{14\}$
- $4(n + 1) = 3(4 + n) \{8\}$
- $5y + 3 = 53 \{10\}$
- $2(x - 1) - 5 = 9 \{8\}$
- $5(y - 2) + 4 = 14 \{4\}$